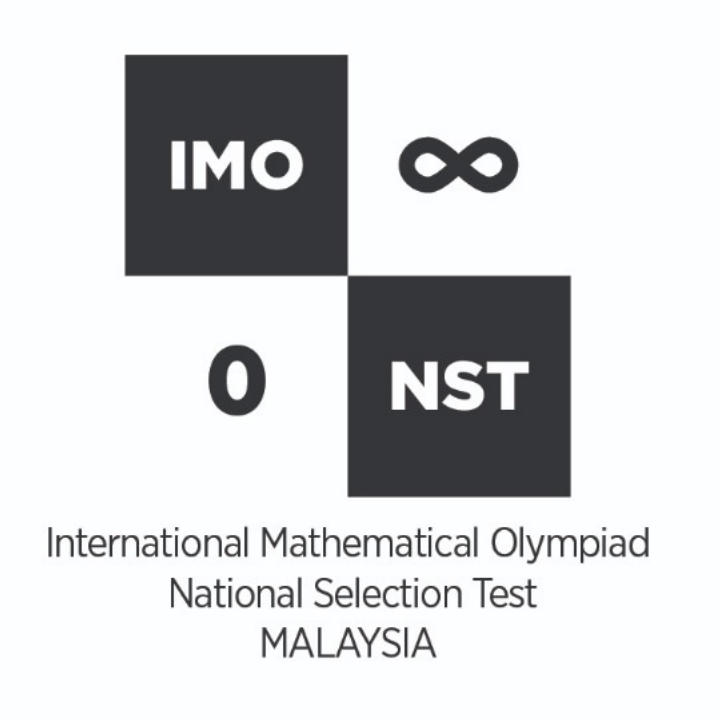
SAMPLE PROBLEMS 1 (IMONST 1)



Malaysia IMO Committee
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1 About IMONST

IMO National Selection Test (IMONST) is a national-level mathematics competition whose objective is to promote mathematical problem solving among Malaysian students, and challenge the top mathematical talents in the country. It is organized by the Malaysia IMO Committee. IMONST is approved by the MoE as the selection process for the Malaysian team for the International Mathematical Olympiad (IMO) 2021.

The IMO is the World Championship Mathematics Competition for High School students and is held annually in a different country. The first IMO was held in 1959 in Romania, with 7 countries participating. It has gradually expanded to over 100 countries from 5 continents.

There are two rounds of IMONST: IMONST 1 is an open round, while IMONST 2 is by invitation only.

This booklet covers some sample problems that are comparable to the difficulty of the IMONST 1 paper.

For more details about IMONST, go to `https://imo-malaysia.org/imonst/` .

Categories

There are three categories in IMONST 1:

1. Primary – advanced primary school students
2. Junior – Form 1 to Form 3 students
3. Senior – Form 4 to Form 6, and pre-university students.

This is the first time that primary school students are involved in IMO selection in Malaysia. Although the IMONST is perhaps too difficult for the average primary student, bear in mind that there are exceptional mathematical talents of a very young age (as an example, one of the Malaysian participants in IMO 2014 was 12 years old). The IMONST aims to identify the young talents so they can be groomed to be part of future IMO teams.

Format of IMONST 1

IMONST is an online, individual, open-book competition. Students are allowed to use any reference and calculating tools, as long as they sit for the competition themselves without any external help. The problems are designed such that it can be solved without using a calculator.

There are 20 questions for each category, divided into 4 parts (A to D). The parts are arranged in increasing order of difficulty. Every correct answer is awarded 1, 2, 3, 4

points for Part A, B, C, D, respectively. No point is deducted for an incorrect answer. The maximum score is 50 points.

For every question, only the answer needs to be provided. The answer to each question is a non-negative integer.

Problems in IMONST 1 are provided in both Bahasa Melayu and English.

Contact Us

Email the IMO Malaysia Committee at `contact@imo-malaysia.org` .

Version

Version 1.2, updated on 16 August 2020.

Part I

Bahasa Melayu

2 Kategori *Primary* & *Junior*

Bahagian A (1 markah setiap soalan)

Soalan 1. Diberi nombor nyata x, y, dan z yang memenuhi persamaan

$$(x-1)^2 + 2(y-2)^2 + 3(z-3)^2 = 0.$$

Tentukan nilai bagi xyz.

Soalan 2. Pertimbangkan jujukan nombor berikut:

$$-2, 4, -6, 8, -10, 12, -14, 16, \ldots .$$

Apakah purata bagi 2020 sebutan terawal bagi jujukan tersebut?

Soalan 3. Cari digit terakhir bagi hasil tambah berikut:

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots + 100^2.$$

Soalan 4. Di kalangan 250 orang pelajar di suatu sekolah, 150 orang telah mengambil bahagian dalam Olimpiad Matematik dan 130 orang telah mengambil bahagian dalam Olimpiad Sains. Setiap pelajar mengambil bahagian dalam sekurang-kurangnya satu Olimpiad. Berapakah bilangan pelajar yang menyertai kedua-dua Olimpiad?

Soalan 5. Titik P terletak di dalam segiempat $ABCD$ supaya segitiga ABP adalah sama sisi. Cari $\angle PCD$ dalam unit darjah.

Bahagian B (2 markah setiap soalan)

Soalan 6. Tentukan integer positif terkecil yang meninggalkan baki 1 apabila dibahagi dengan 2, baki 2 apabila dibahagi dengan 3, baki 3 apabila dibahagi dengan 4, dan baki 4 apabila dibahagi dengan 5.

Soalan 7. Jika setiap murid lelaki dalam sebuah kelas membeli satu donat dan setiap murid perempuan membeli satu karipap, mereka akan membelanjakan kurang satu sen berbanding jika setiap murid lelaki membeli satu karipap dan setiap murid perempuan membeli satu donat. Diketahui bahawa bilangan murid lelaki melebihi bilangan murid perempuan. Cari beza antara bilangan murid lelaki dan bilangan murid perempuan.

Soalan 8. Kita mempunyai enam batang dengan ukuran panjang berikut: 1cm, 3cm, 5cm, 7cm, 11cm, dan 13cm. Berapakah bilangan segitiga berbeza yang boleh dibentuk menggunakan mana-mana tiga batang tersebut sebagai sisi?

Soalan 9. Hasil tambah digit bagi 2020 ialah 4. Berapakah bilangan integer empat digit (termasuk 2020) yang mempunyai hasil tambah digit bersamaan 4?

Soalan 10. Suatu jujukan $a_1, a_2, a_3, \ldots$ ditakrifkan sebagai

$$a_1 = 2, \quad a_2 = 5, \quad \text{dan} \quad a_{n+2} = \frac{1 + a_{n+1}}{a_n} \quad \text{bagi semua } n \geq 1.$$

Cari nilai a_{123}.

Bahagian C (3 markah setiap soalan)

Soalan 11. Tentukan bilangan integer genap antara 4000 dengan 9000 dengan semua empat digitnya berbeza.

Soalan 12. Bagi suatu integer positif n, andaikan $E(n)$ mewakili hasil tambah digit-digit genap bagi n. Contohnya, $E(832) = 8 + 2 = 10$. Cari nilai bagi

$$E(1) + E(2) + E(3) + E(4) + \cdots + E(1000).$$

Soalan 13. Dalam jujukan berikut, setiap integer positif k berulang sebanyak k kali:

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots .$$

Tentukan sebutan ke-2020 bagi jujukan tersebut.

Soalan 14. Beberapa integer positif mempunyai hasil tambah 22. Apakah hasil darab terbesar yang mungkin bagi integer-integer tersebut?

Soalan 15. Dalam suatu segitiga PQR, $\angle PQR = \angle PRQ = 70^\circ$. Titik S dan T adalah masing-masing terletak pada sisi PQ dan PR, sehinggakan $\angle RQT = 55^\circ$ dan $\angle QRS = 40^\circ$. Cari $\angle PST$ dalam unit darjah.

Bahagian D (4 markah setiap soalan)

Soalan 16. Suatu segitiga mempunyai sisi-sisi dengan panjang 11, 15, dan k, dengan k suatu integer. Berapakah bilangan nilai k yang menghasilkan segitiga bersudut cakah? (Segitiga bersudut cakah mempunyai satu sudut dalaman yang lebih besar daripada $90°$.)

Soalan 17. Nombor N adalah integer terbesar yang mempunyai sifat-sifat berikut:

(i) N adalah gandaan 8;

(ii) semua digit bagi N adalah berbeza.

Apakah tiga digit terakhir bagi N?

Soalan 18. Integer-integer positif ditulis secara berturutan bermula daripada 1:

$$123456789101112131415161718192021\cdots$$

Digit yang ke-17 yang ditulis adalah 3 (dibariskan). Apakah digit yang ke-2020 yang ditulis?

Soalan 19. Diberi suatu trapezium $ABCD$ dengan luas 100. Titik tengah bagi AB, BC, CD, dan DA masing-masing adalah K, L, M, dan N. Cari luas sisiempat $KLMN$.

Soalan 20. Cari integer positif paling kecil yang meninggalkan baki 1 apabila dibahagi dengan 4, dan tidak boleh diungkapkan sebagai $a^2 + b^2$, dengan a dan b integer.

3 Kategori *Senior*

Bahagian A (1 markah setiap soalan)

Soalan 1. Tentukan integer positif terkecil yang meninggalkan baki 1 apabila dibahagi dengan 2, baki 2 apabila dibahagi dengan 3, baki 3 apabila dibahagi dengan 4, dan baki 4 apabila dibahagi dengan 5.

Soalan 2. Jika setiap murid lelaki dalam sebuah kelas membeli satu donat dan setiap murid perempuan membeli satu karipap, mereka akan membelanjakan kurang satu sen berbanding jika setiap murid lelaki membeli satu karipap dan setiap murid perempuan membeli satu donat. Diketahui bahawa bilangan murid lelaki melebihi bilangan murid perempuan. Cari beza antara bilangan murid lelaki dan bilangan murid perempuan.

Soalan 3. Kita mempunyai enam batang dengan ukuran panjang berikut: 1cm, 3cm, 5cm, 7cm, 11cm, dan 13cm. Berapakah bilangan segitiga berbeza yang boleh dibentuk menggunakan mana-mana tiga batang tersebut sebagai sisi?

Soalan 4. Hasil tambah digit bagi 2020 ialah 4. Berapakah bilangan integer empat digit (termasuk 2020) yang mempunyai hasil tambah digit bersamaan 4?

Soalan 5. Suatu jujukan $a_1, a_2, a_3, \ldots$ ditakrifkan sebagai

$$a_1 = 2, \quad a_2 = 5, \quad \text{dan} \quad a_{n+2} = \frac{1 + a_{n+1}}{a_n} \quad \text{bagi semua } n \geq 1.$$

Cari nilai a_{123}.

Bahagian B (2 markah setiap soalan)

Soalan 6. Tentukan bilangan integer genap antara 4000 dengan 9000 dengan semua empat digitnya berbeza.

Soalan 7. Bagi suatu integer positif n, andaikan $E(n)$ mewakili hasil tambah digit-digit genap bagi n. Contohnya, $E(832) = 8 + 2 = 10$. Cari nilai bagi

$$E(1) + E(2) + E(3) + E(4) + \cdots + E(1000).$$

Soalan 8. Dalam jujukan berikut, setiap integer positif k berulang sebanyak k kali:

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots .$$

Tentukan sebutan ke-2020 bagi jujukan tersebut.

Soalan 9. Beberapa integer positif mempunyai hasil tambah 22. Apakah hasil darab terbesar yang mungkin bagi integer-integer tersebut?

Soalan 10. Dalam suatu segitiga PQR, $\angle PQR = \angle PRQ = 70^\circ$. Titik S dan T adalah masing-masing terletak pada sisi PQ dan PR, sehinggakan $\angle RQT = 55^\circ$ dan $\angle QRS = 40^\circ$. Cari $\angle PST$ dalam unit darjah.

Bahagian C (3 markah setiap soalan)

Soalan 11. Suatu segitiga mempunyai sisi-sisi dengan panjang 11, 15, dan k, dengan k suatu integer. Berapakah bilangan nilai k yang menghasilkan segitiga bersudut cakah? (Segitiga bersudut cakah mempunyai satu sudut dalaman yang lebih besar daripada $90°$.)

Soalan 12. Nombor N adalah integer terbesar yang mempunyai sifat-sifat berikut:

(i) N adalah gandaan 8;

(ii) semua digit bagi N adalah berbeza.

Apakah tiga digit terakhir bagi N?

Soalan 13. Integer-integer positif ditulis secara berturutan bermula daripada 1:

$$123456789101112\underline{1}3141516171819202 1\cdots$$

Digit yang ke-17 yang ditulis adalah 3 (dibariskan). Apakah digit yang ke-2020 yang ditulis?

Soalan 14. Diberi suatu trapezium $ABCD$ dengan luas 100. Titik tengah bagi AB, BC, CD, dan DA masing-masing adalah K, L, M, dan N. Cari luas sisiempat $KLMN$.

Soalan 15. Cari integer positif paling kecil yang meninggalkan baki 1 apabila dibahagi dengan 4, dan tidak boleh diungkapkan sebagai $a^2 + b^2$, dengan a dan b integer.

Bahagian D (4 markah setiap soalan)

Soalan 16. Tentukan bilangan sifar di penghujung 2020!.

Soalan 17. Cari integer terbesar N sehinggakan pernyataan berikut adalah salah:

Suatu segiempat sama boleh dipecahkan kepada N segiempat sama yang lebih kecil (tidak semestinya bersaiz sama) tanpa meninggalkan baki.

Soalan 18. Tiga bulatan dengan panjang jejari 45, 45, dan 72 adalah bertangen sesama sendiri secara luaran. Suatu bulatan yang lain pula adalah bertangen secara luaran dengan ketiga-tiga bulatan tersebut. Apakah jejari bagi bulatan keempat tersebut?

Soalan 19. Dua jujukan berikut mengikut pola lazim yang boleh dihuraikan dengan cara yang mudah:

$$1, 2, 4, 6, 10, 12, A, \ldots ;$$
$$4, 8, 32, 128, 2048, 8192, B, \ldots .$$

Cari nilai bagi $\dfrac{B}{A}$.

Soalan 20. Cari hasil darab bagi semua penyelesaian nyata kepada persamaan

$$(x^2 - 7x + 11)^{x^2-7x+6} = 1.$$

Part II
English

4 Primary & Junior Category

Part A (1 point each)

Problem 1. Given real numbers x, y, and z that fulfill the equation

$$(x-1)^2 + 2(y-2)^2 + 3(z-3)^2 = 0.$$

Determine the value of xyz.

Problem 2. Consider the following sequence of numbers

$$-2, 4, -6, 8, -10, 12, -14, 16, \ldots .$$

What is the average of the first 2020 terms of the sequence?

Problem 3. Find the last digit of the following sum:

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots + 100^2.$$

Problem 4. Among 250 students at a school, 150 have taken part in the Mathematics Olympiad and 130 in the Science Olympiad. Each student participates in at least one Olympiad. How many students have participated in both Olympiads?

Problem 5. A point P is inside square $ABCD$ such that triangle ABP is equilateral. Find $\angle PCD$ in degrees.

Part B (2 points each)

Problem 6. Find the smallest positive integer which leaves remainder 1 when divided by 2, remainder 2 when divided by 3, remainder 3 when divided by 4, and remainder 4 when divided by 5.

Problem 7. If every boy in a class buys a doughnut and every girl buys a karipap, then they will spend one sen less than if every boy buys a karipap and every girl buys a doughnut. We know that there are more boys than girls. Find the difference between the number of boys and the number of girls.

Problem 8. We have six sticks of the following lengths: 1cm, 3cm, 5cm, 7cm, 11cm, and 13cm. How many different triangles can be made using any three of these sticks as sides?

Problem 9. The digit sum of 2020 is 4. How many four-digit integers (including 2020) have digit sum equal to 4?

Problem 10. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2, \quad a_2 = 5, \quad \text{and} \quad a_{n+2} = \frac{1 + a_{n+1}}{a_n} \quad \text{for all } n \geq 1.$$

Find the value of a_{123}.

Part C (3 points each)

Problem 11. Determine the number of even integers between 4000 and 9000 with all four digits different.

Problem 12. For any positive integer n, let $E(n)$ denote the sum of the even digits of n. For example, $E(832) = 8 + 2 = 10$. Find the value of

$$E(1) + E(2) + E(3) + E(4) + \cdots + E(1000).$$

Problem 13. In the following sequence, every positive integer k appears k times:

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots .$$

Determine the 2020th term of the sequence.

Problem 14. Some positive integers have sum 22. What is the maximum possible product of these integers?

Problem 15. In a triangle PQR, $\angle PQR = \angle PRQ = 70^\circ$. Points S and T are on sides PQ and PR respectively, so that $\angle RQT = 55^\circ$ and $\angle QRS = 40^\circ$. Find $\angle PST$ in degrees.

Part D (4 points each)

Problem 16. A triangle have sides of lengths 11, 15, and k, where k is an integer. For how many values of k is the triangle obtuse? (An obtuse triangle has one interior angle greater than $90°$.)

Problem 17. The number N is the largest integer with these properties:

(i) N is a multiple of 8;

(ii) all digits of N are different.

What are the last three digits of N?

Problem 18. The positive integers are written in order beginning with 1:

$$123456789101112\underline{1}31415161718192021\cdots$$

The 17th digit that gets written is 3 (underlined). What is the 2020th digit that gets written?

Problem 19. Given a trapezium $ABCD$ with area 100. The midpoints of AB, BC, CD, and DA are K, L, M, and N respectively. Find the area of quadrilateral $KLMN$.

Problem 20. Find the smallest positive integer that leaves remainder 1 when divided by 4, and cannot be expressed as $a^2 + b^2$, where a and b are integers.

5 Senior Category

Part A (1 point each)

Problem 1. Find the smallest positive integer which leaves remainder 1 when divided by 2, remainder 2 when divided by 3, remainder 3 when divided by 4, and remainder 4 when divided by 5.

Problem 2. If every boy in a class buys a doughnut and every girl buys a karipap, then they will spend one sen less than if every boy buys a karipap and every girl buys a doughnut. We know that there are more boys than girls. Find the difference between the number of boys and the number of girls.

Problem 3. We have six sticks of the following lengths: 1cm, 3cm, 5cm, 7cm, 11cm, and 13cm. How many different triangles can be made using any three of these sticks as sides?

Problem 4. The digit sum of 2020 is 4. How many four-digit integers (including 2020) have digit sum equal to 4?

Problem 5. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2, \quad a_2 = 5, \quad \text{and} \quad a_{n+2} = \frac{1 + a_{n+1}}{a_n} \quad \text{for all } n \geq 1.$$

Find the value of a_{123}.

Part B (2 points each)

Problem 6. Determine the number of even integers between 4000 and 9000 with all four digits different.

Problem 7. For any positive integer n, let $E(n)$ denote the sum of the even digits of n. For example, $E(832) = 8 + 2 = 10$. Find the value of

$$E(1) + E(2) + E(3) + E(4) + \cdots + E(1000).$$

Problem 8. In the following sequence, every positive integer k appears k times:

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots .$$

Determine the 2020th term of the sequence.

Problem 9. Some positive integers have sum 22. What is the maximum possible product of these integers?

Problem 10. In a triangle PQR, $\angle PQR = \angle PRQ = 70^\circ$. Points S and T are on sides PQ and PR respectively, so that $\angle RQT = 55^\circ$ and $\angle QRS = 40^\circ$. Find $\angle PST$ in degrees.

Part C (3 points each)

Problem 11. A triangle have sides of lengths 11, 15, and k, where k is an integer. For how many values of k is the triangle obtuse? (An obtuse triangle has one interior angle greater than $90°$.)

Problem 12. The number N is the largest integer with these properties:

(i) N is a multiple of 8;

(ii) all digits of N are different.

What are the last three digits of N?

Problem 13. The positive integers are written in order beginning with 1:

$$123456789101111213\underline{1}41516171819202 1\cdots$$

The 17th digit that gets written is 3 (underlined). What is the 2020th digit that gets written?

Problem 14. Given a trapezium $ABCD$ with area 100. The midpoints of AB, BC, CD, and DA are K, L, M, and N respectively. Find the area of quadrilateral $KLMN$.

Problem 15. Find the smallest positive integer that leaves remainder 1 when divided by 4, and cannot be expressed as $a^2 + b^2$, where a and b are integers.

Part D (4 points each)

Problem 16. Determine the number of zeros at the end of 2020!.

Problem 17. Find the largest integer N such that the following statement is false:

A square can be dissected into N smaller squares (not necessarily the same size) without any remainder.

Problem 18. Three circles with radii 45, 45, and 72 are externally tangent to each other. Another circle is externally tangent to these three circles. What is the radius of the fourth circle?

Problem 19. Each of the following two sequences follows a regular pattern that can be explained by a simple rule:

$$1, 2, 4, 6, 10, 12, A, \ldots ;$$
$$4, 8, 32, 128, 2048, 8192, B, \ldots .$$

Find the value of $\dfrac{B}{A}$.

Problem 20. Find the product of all real solutions to the equation

$$(x^2 - 7x + 11)^{x^2-7x+6} = 1.$$

Part III

Jawapan/Answers

6 Kategori *Primary* & *Junior*/ Primary & Junior Category

Problem 1. 6

Problem 2. 1

Problem 3. 0

Problem 4. 30

Problem 5. 15

Problem 6. 59

Problem 7. 1

Problem 8. 5

Problem 9. 20

Problem 10. 3

Problem 11. 1232

Problem 12. 6000

Problem 13. 64

Problem 14. 2916

Problem 15. 35

Problem 16. 13

Problem 17. 120

Problem 18. 7

Problem 19. 50

Problem 20. 21

7 Kategori *Senior* / Senior Category

Problem 1. 59

Problem 2. 1

Problem 3. 5

Problem 4. 20

Problem 5. 3

Problem 6. 1232

Problem 7. 6000

Problem 8. 64

Problem 9. 2916

Problem 10. 35

Problem 11. 13

Problem 12. 120

Problem 13. 7

Problem 14. 50

Problem 15. 21

Problem 16. 503

Problem 17. 5

Problem 18. 8

Problem 19. 8192

Problem 20. 720