SAMPLE PROBLEMS 2
(IMONST 1)

International Mathematical Olympiad
National Selection Test
MALAYSIA

Malaysia IMO Committee
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Contents

1 About IMONST

IMO National Selection Test (IMONST) is a national-level mathematics competition whose objective is to promote mathematical problem solving among Malaysian students, and challenge the top mathematical talents in the country. It is organized by the Malaysia IMO Committee. IMONST is approved by the MoE as the selection process for the Malaysian team for the International Mathematical Olympiad (IMO) 2021.

The IMO is the World Championship Mathematics Competition for High School students and is held annually in a different country. The first IMO was held in 1959 in Romania, with 7 countries participating. It has gradually expanded to over 100 countries from 5 continents.

There are two rounds of IMONST: IMONST 1 is an open round, while IMONST 2 is by invitation only.

This booklet covers some sample problems that are comparable to the difficulty of the IMONST 1 paper.

For more details about IMONST, go to `https://imo-malaysia.org/imonst/` .

Categories

There are three categories in IMONST 1:

1. Primary – advanced primary school students
2. Junior – Form 1 to Form 3 students
3. Senior – Form 4 to Form 6, and pre-university students.

This is the first time that primary school students are involved in IMO selection in Malaysia. Although the IMONST is perhaps too difficult for the average primary student, bear in mind that there are exceptional mathematical talents of a very young age (as an example, one of the Malaysian participants in IMO 2014 was 12 years old). The IMONST aims to identify the young talents so they can be groomed to be part of future IMO teams.

Format of IMONST 1

IMONST is an online, individual, open-book competition. Students are allowed to use any reference and calculating tools, as long as they sit for the competition themselves without any external help. The problems are designed such that it can be solved without using a calculator.

There are 20 questions for each category, divided into 4 parts (A to D). The parts are arranged in increasing order of difficulty. Every correct answer is awarded 1, 2, 3, 4 points for Part A, B, C, D, respectively. No point is deducted for an incorrect answer. The maximum score is 50 points.

For every question, only the answer needs to be provided. The answer to each question is a non-negative integer.

Problems in IMONST 1 are provided in both Bahasa Melayu and English.

Contact Us

Email the IMO Malaysia Committee at `contact@imo-malaysia.org` .

Version

Version 1.0, updated on 16 August 2020.

Part I

Bahasa Melayu

2 Kategori *Primary*

Bahagian A (1 markah setiap soalan)

Soalan 1. Ali melontar sebiji dadu sebanyak empat kali. Hasil tambah semua nombor yang dia peroleh ialah 23. Berapa kalikah Ali melontarkan nombor 5?

Soalan 2. Seorang kanak-kanak lelaki bernama Bala mempunyai bilangan adik-beradik lelaki yang sama dengan bilangan adik-beradik perempuan. Kakaknya Chita pula mempunyai dua kali lebih ramai adik-beradik lelaki berbanding adik-beradik perempuan. Berapa bilangan adik-beradik kesemuanya?

Soalan 3. Suatu nombor dianggap *baik* jika ia mempunyai 3 digit, dan kesemua digitnya berbeza. Apakah beza antara nombor baik yang paling besar dan yang paling kecil?

Soalan 4. Berapakah bilangan nombor yang mempunyai hasil tambah digit-digitnya bersamaan 15 dan hasil darab digit-digitnya bersamaan 5?

Soalan 5. Diberi segiempat sama $ABCD$ dengan panjang sisi 4, dan suatu titik E di luar segiempat tersebut. Diketahui bahawa segitiga ABE tidak bertindan dengan segiempat tersebut, dan perimeternya adalah sama dengan perimeter bagi segiempat tersebut. Apakah perimeter bagi pentagon $AEBCD$?

Bahagian B (2 markah setiap soalan)

Soalan 6. Sebuah kuboid mempunyai panjang 3, lebar 3 dan tinggi 23. Ia mempunyai luas permukaan yang sama dengan sebuah kubus. Berapakah panjang sisi kubus tersebut?

Soalan 7. Jika kita menulis 2020 sebagai hasil tambah 5 integer berturutan, apakah integer yang paling kecil?

Soalan 8. Cari digit terakhir bagi $4^4 + 5^5 + 6^6 + 7^7$.

Soalan 9. Hasil darab dua integer positif M and N adalah 50 kali hasil tambah kedua-dua nombor tersebut dan 75 kali beza kedua-dua nombor tersebut. Cari $M + N$.

Soalan 10. Tentukan bilangan pasangan integer positif (x, y) yang memenuhi ketaksamaan $5x + 3y \leq 53$.

Nota: Pasangan $(1, 2)$ dan $(2, 1)$ dianggap sebagai pasangan berbeza.

Bahagian C (3 markah setiap soalan)

Soalan 11. Diberi bahawa

$$4^{4^n} = 2^{2^{3^{2^2}}}.$$

Cari nilai n.

Soalan 12. Tentukan integer paling besar supaya tiada digit yang digunakan lebih daripada sekali, dan tiada dua digit bersebelahan yang mempunyai beza 1.

Soalan 13. Diberi suatu nombor enam digit N. Berapakah bilangan nombor tujuh digit yang wujud supaya jika salah satu digitnya dibuang, hasilnya ialah N?

Soalan 14. Diberi sisiempat $ABCD$ dengan $AB = BC = CD$, $\angle ABC = 90°$, dan $\angle BCD = 60°$. Cari $\angle ADC$ dalam unit darjah.

Soalan 15. Suatu kelab matematik mempunyai 7 ahli. Penasihat kelab tersebut mahu memilih sekurang-kurangnya 4 ahli untuk mewakili kelab tersebut dalam suatu pertandingan matematik. Berapakah bilangan cara untuk membuat pemilihan tersebut?

Bahagian D (4 markah setiap soalan)

Soalan 16. Di sebuah kedai buah-buahan, buah oren tidak dijual secara berasingan; ia hanya dijual dalam bungkusan kecil (3 biji oren) atau dalam bungkusan besar (10 biji oren). Contohnya, kita boleh membeli 13 biji oren (satu bungkusan kecil dan satu bungkusan besar) atau 15 biji oren (5 bungkusan kecil), tetapi kita tidak boleh membeli tepat 14 biji oren. Berapakah bilangan oren maksimum yang tidak boleh dibeli dengan tepat?

Soalan 17. Cari integer positif k yang memenuhi

$$(3^3)(6^6)(15^{15})(21^{21})(k^k) = (5^5)(7^7)(10^{10})(14^{14})(27^{27}).$$

Soalan 18. Tentukan bilangan integer k dengan $1 \leq k \leq 2020$ supaya hasil tambah

$$1 + 2 + 3 + \cdots + k$$

boleh bahagi dengan 5.

Soalan 19. Diberi suatu segitiga ABC. Titik D terletak pada sisi BC. Titik E juga terletak pada sisi BC dengan AE membahagi dua $\angle CAD$. Jika $\angle AEB = 60°$, apakah $\angle ACB + \angle ADB$ dalam unit darjah?

Soalan 20. Bagi sebarang dua integer positif a dan b dengan bilangan digit yang sama, operasi $\langle a, b \rangle$ melambangkan hasil tambah bagi hasil darab digit-digit a and b yang sepadan. Sebagai contoh, $\langle 1234, 5678 \rangle = (1 \times 5) + (2 \times 6) + (3 \times 7) + (4 \times 8) = 70$.

Tentukan nilai bagi

$$\Big\langle \underbrace{123123123\cdots}_{\text{2020 digit}}, \underbrace{123412341234\cdots}_{\text{2020 digit}} \Big\rangle.$$

3 Kategori *Junior*

Bahagian A (1 markah setiap soalan)

Soalan 1. Sebuah kuboid mempunyai panjang 3, lebar 3 dan tinggi 23. Ia mempunyai luas permukaan yang sama dengan sebuah kubus. Berapakah panjang sisi kubus tersebut?

Soalan 2. Jika kita menulis 2020 sebagai hasil tambah 5 integer berturutan, apakah integer yang paling kecil?

Soalan 3. Cari digit terakhir bagi $4^4 + 5^5 + 6^6 + 7^7$.

Soalan 4. Hasil darab dua integer positif M and N adalah 50 kali hasil tambah kedua-dua nombor tersebut dan 75 kali beza kedua-dua nombor tersebut. Cari $M + N$.

Soalan 5. Tentukan bilangan pasangan integer positif (x, y) yang memenuhi ketaksamaan $5x + 3y \leq 53$.

Nota: Pasangan $(1, 2)$ dan $(2, 1)$ dianggap sebagai pasangan berbeza.

Bahagian B (2 markah setiap soalan)

Soalan 6. Diberi bahawa

$$4^{4^n} = 2^{2^{3^{2^2}}}.$$

Cari nilai n.

Soalan 7. Tentukan integer paling besar supaya tiada digit yang digunakan lebih daripada sekali, dan tiada dua digit bersebelahan yang mempunyai beza 1.

Soalan 8. Diberi suatu nombor enam digit N. Berapakah bilangan nombor tujuh digit yang wujud supaya jika salah satu digitnya dibuang, hasilnya ialah N?

Soalan 9. Diberi sisiempat $ABCD$ dengan $AB = BC = CD$, $\angle ABC = 90°$, dan $\angle BCD = 60°$. Cari $\angle ADC$ dalam unit darjah.

Soalan 10. Suatu kelab matematik mempunyai 7 ahli. Penasihat kelab tersebut mahu memilih sekurang-kurangnya 4 ahli untuk mewakili kelab tersebut dalam suatu pertandingan matematik. Berapakah bilangan cara untuk membuat pemilihan tersebut?

Bahagian C (3 markah setiap soalan)

Soalan 11. Di sebuah kedai buah-buahan, buah oren tidak dijual secara berasingan; ia hanya dijual dalam bungkusan kecil (3 biji oren) atau dalam bungkusan besar (10 biji oren). Contohnya, kita boleh membeli 13 biji oren (satu bungkusan kecil dan satu bungkusan besar) atau 15 biji oren (5 bungkusan kecil), tetapi kita tidak boleh membeli tepat 14 biji oren. Berapakah bilangan oren maksimum yang tidak boleh dibeli dengan tepat?

Soalan 12. Cari integer positif k yang memenuhi

$$(3^3)(6^6)(15^{15})(21^{21})(k^k) = (5^5)(7^7)(10^{10})(14^{14})(27^{27}).$$

Soalan 13. Tentukan bilangan integer k dengan $1 \leq k \leq 2020$ supaya hasil tambah

$$1 + 2 + 3 + \cdots + k$$

boleh bahagi dengan 5.

Soalan 14. Diberi suatu segitiga ABC. Titik D terletak pada sisi BC. Titik E juga terletak pada sisi BC dengan AE membahagi dua $\angle CAD$. Jika $\angle AEB = 60°$, apakah $\angle ACB + \angle ADB$ dalam unit darjah?

Soalan 15. Bagi sebarang dua integer positif a dan b dengan bilangan digit yang sama, operasi $\langle a, b \rangle$ melambangkan hasil tambah bagi hasil darab digit-digit a and b yang sepadan. Sebagai contoh, $\langle 1234, 5678 \rangle = (1 \times 5) + (2 \times 6) + (3 \times 7) + (4 \times 8) = 70$.

Tentukan nilai bagi

$$\Big\langle \underbrace{123123123\cdots}_{\text{2020 digit}}, \underbrace{123412341234\cdots}_{\text{2020 digit}} \Big\rangle.$$

Bahagian D (4 markah setiap soalan)

Soalan 16. Pertimbangkan jadual berikut:

$$\begin{array}{ccccc} 1 & & & & \\ 2 & 3 & & & \\ 4 & 5 & 6 & & \\ 7 & 8 & 9 & 10 & \\ 11 & 12 & 13 & 14 & 15 \\ \vdots & & & & \ddots \end{array}$$

Apakah nombor yang berada tepat di bawah nombor 2020?

Soalan 17. Poligon sekata dengan 20 sisi

$$ABCDEFGHIJKLMNPQRSTU$$

mempunyai pusat O. Cari $\angle IMO + \angle NST$ dalam unit darjah.

Soalan 18. Tentukan bilangan integer n yang memenuhi syarat-syarat berikut:

1. $1 \leq n \leq 1000$;
2. hasil tambah digit bagi n adalah ganjil;
3. hasil tambah digit bagi $(n+1)$ adalah ganjil.

Soalan 19. Apakah nombor kuasa dua sempurna dengan empat digit yang bermula dengan dua digit yang sama, dan berakhir dengan dua digit yang sama?

Soalan 20. Terdapat 99 orang pelajar (namakan mereka sebagai Pelajar 1, Pelajar 2, dan seterusnya) dan 99 kerusi mengelilingi sebuah meja bulat. Berapakah bilangan cara menyusun kedudukan pelajar-pelajar tersebut supaya bagi sebarang dua pelajar yang duduk bersebelahan, beza antara nombor mereka tidak melebihi 2?

4 Kategori *Senior*

Bahagian A (1 markah setiap soalan)

Soalan 1. Diberi bahawa

$$4^{4^n} = 2^{2^{3^{2^2}}}.$$

Cari nilai n.

Soalan 2. Tentukan integer paling besar supaya tiada digit yang digunakan lebih daripada sekali, dan tiada dua digit bersebelahan yang mempunyai beza 1.

Soalan 3. Diberi suatu nombor enam digit N. Berapakah bilangan nombor tujuh digit yang wujud supaya jika salah satu digitnya dibuang, hasilnya ialah N?

Soalan 4. Diberi sisiempat $ABCD$ dengan $AB = BC = CD$, $\angle ABC = 90^\circ$, dan $\angle BCD = 60^\circ$. Cari $\angle ADC$ dalam unit darjah.

Soalan 5. Suatu kelab matematik mempunyai 7 ahli. Penasihat kelab tersebut mahu memilih sekurang-kurangnya 4 ahli untuk mewakili kelab tersebut dalam suatu pertandingan matematik. Berapakah bilangan cara untuk membuat pemilihan tersebut?

Bahagian B (2 markah setiap soalan)

Soalan 6. Di sebuah kedai buah-buahan, buah oren tidak dijual secara berasingan; ia hanya dijual dalam bungkusan kecil (3 biji oren) atau dalam bungkusan besar (10 biji oren). Contohnya, kita boleh membeli 13 biji oren (satu bungkusan kecil dan satu bungkusan besar) atau 15 biji oren (5 bungkusan kecil), tetapi kita tidak boleh membeli tepat 14 biji oren. Berapakah bilangan oren maksimum yang tidak boleh dibeli dengan tepat?

Soalan 7. Cari integer positif k yang memenuhi

$$(3^3)(6^6)(15^{15})(21^{21})(k^k) = (5^5)(7^7)(10^{10})(14^{14})(27^{27}).$$

Soalan 8. Tentukan bilangan integer k dengan $1 \leq k \leq 2020$ supaya hasil tambah

$$1 + 2 + 3 + \cdots + k$$

boleh bahagi dengan 5.

Soalan 9. Diberi suatu segitiga ABC. Titik D terletak pada sisi BC. Titik E juga terletak pada sisi BC dengan AE membahagi dua $\angle CAD$. Jika $\angle AEB = 60°$, apakah $\angle ACB + \angle ADB$ dalam unit darjah?

Soalan 10. Bagi sebarang dua integer positif a dan b dengan bilangan digit yang sama, operasi $\langle a, b\rangle$ melambangkan hasil tambah bagi hasil darab digit-digit a and b yang sepadan. Sebagai contoh, $\langle 1234, 5678\rangle = (1 \times 5) + (2 \times 6) + (3 \times 7) + (4 \times 8) = 70$.

Tentukan nilai bagi

$$\Big\langle \underbrace{123123123\cdots}_{\text{2020 digit}}, \underbrace{123412341234\cdots}_{\text{2020 digit}} \Big\rangle.$$

Bahagian C (3 markah setiap soalan)

Soalan 11. Pertimbangkan jadual berikut:

$$\begin{array}{lllll} 1 & & & & \\ 2 & 3 & & & \\ 4 & 5 & 6 & & \\ 7 & 8 & 9 & 10 & \\ 11 & 12 & 13 & 14 & 15 \\ \vdots & & & & & \ddots \end{array}$$

Apakah nombor yang berada tepat di bawah nombor 2020?

Soalan 12. Poligon sekata dengan 20 sisi

$$ABCDEFGHIJKLMNPQRSTU$$

mempunyai pusat O. Cari $\angle IMO + \angle NST$ dalam unit darjah.

Soalan 13. Tentukan bilangan integer n yang memenuhi syarat-syarat berikut:

1. $1 \leq n \leq 1000$;
2. hasil tambah digit bagi n adalah ganjil;
3. hasil tambah digit bagi $(n + 1)$ adalah ganjil.

Soalan 14. Apakah nombor kuasa dua sempurna dengan empat digit yang bermula dengan dua digit yang sama, dan berakhir dengan dua digit yang sama?

Soalan 15. Terdapat 99 orang pelajar (namakan mereka sebagai Pelajar 1, Pelajar 2, dan seterusnya) dan 99 kerusi mengelilingi sebuah meja bulat. Berapakah bilangan cara menyusun kedudukan pelajar-pelajar tersebut supaya bagi sebarang dua pelajar yang duduk bersebelahan, beza antara nombor mereka tidak melebihi 2?

Bahagian D (4 markah setiap soalan)

Soalan 16. Nombor $\overline{2a9b} = (2 \times 1000) + (a \times 100) + (9 \times 10) + b$ adalah hasil darab dua integer 2^a dan 9^b. Berapakah bilangan nilai berbeza bagi $a + b$ yang wujud?

Soalan 17. Diberi segiempat sama $ABCD$. Titik K membahagikan sisi AB dengan nisbah $AK : KB = 2 : 1$. Titik L membahagi pepenjuru AC dengan nisbah $AL : LC = 5 : 1$. Cari $\angle KLD$ dalam unit darjah.

Soalan 18. Jika 2500 ditambah dengan suatu nombor perdana p, kita memperoleh suatu nombor kuasa dua sempurna. Apakah p?

Soalan 19. Tentukan bilangan pasangan integer bukan sifar (m, n) yang memenuhi persamaan berikut:

$$(m^2 + n)(m + n^2) = (m + n)^3.$$

Nota: Pasangan $(1, 2)$ dan $(2, 1)$ dianggap sebagai pasangan berbeza.

Soalan 20. Terdapat 31 orang pelajar di dalam sebuah kelas; mereka dinamakan sebagai Pelajar 1, Pelajar 2, Pelajar 3, dan seterusnya. Seorang guru menulis satu nombor di papan putih. Kemudian, setiap pelajar membuat satu pernyataan mengenai nombor tersebut:

Pelajar 1 berkata, "nombor itu boleh bahagi dengan 1."

Kemudian, Pelajar 2 berkata, "nombor itu boleh bahagi dengan 2."

Kemudian, Pelajar 3 berkata, "nombor itu boleh bahagi dengan 3."

$\vdots$

Kemudian, Pelajar 31 berkata, "nombor itu boleh bahagi dengan 31."

Akhirnya, guru itu berkata, "Kamu semua telah membuat pernyataan yang benar, kecuali dua orang pelajar. Kedua-dua pelajar ini membuat pernyataan mereka secara berturutan."

Pelajar manakah yang membuat pernyataan tidak benar yang pertama?

Part II

English

5 Primary Category

Part A (1 point each)

Problem 1. Ali rolled a dice four times. The sum of all the numbers he obtained is 23. How many times did Ali roll number 5?

Problem 2. A boy named Bala has the same number of brothers and sisters. His sister Chita has twice as many brothers as sisters. How many siblings are there?

Problem 3. A number is *good* if it has 3 digits, and the digits are all different. What is the difference between the largest and the smallest good numbers?

Problem 4. How many numbers are there that has the sum of the digits equal to 15 and the product of the digits equal to 5?

Problem 5. Given a square $ABCD$ with side length 4, and a point E outside the square. We know that triangle ABE does not intersect the square, and its perimeter is equal to the perimeter of the square. What is the perimeter of the pentagon $AEBCD$?

Part B (2 points each)

Problem 6. A cuboid has length 3, width 3, and height 23. It has the same surface area as a cube. What is the side length of the cube?

Problem 7. If we write 2020 as a sum of 5 consecutive integers, what is the smallest integer?

Problem 8. Find the last digit of $4^4 + 5^5 + 6^6 + 7^7$.

Problem 9. The product of two positive integers M and N is 50 times their sum and 75 times their difference. Find $M + N$.

Problem 10. Determine the number of pairs of positive integers (x, y) that satisfy the inequality $5x + 3y \leq 53$.

Note: The pairs $(1, 2)$ and $(2, 1)$ are considered distinct pairs.

Part C (3 points each)

Problem 11. Given that

$$4^{4^n} = 2^{2^{3^{2^2}}}.$$

Find n.

Problem 12. Determine the greatest integer such that no digit appears more than once, and no two neighbouring digits have difference 1.

Problem 13. Given a six-digit number N. How many seven-digit numbers are there such that when we remove one of its digits, the result is N?

Problem 14. Given a quadrilateral $ABCD$ with $AB = BC = CD$, $\angle ABC = 90°$, and $\angle BCD = 60°$. Find $\angle ADC$ in degrees.

Problem 15. A math club has 7 members. The club advisor wanted to choose at least 4 members to represent the club in a math contest. How many ways are there to do so?

Part D (4 points each)

Problem 16. In a fruit shop, oranges are not sold individually; they are sold either in small packs (3 oranges) or in large packs (10 oranges). For example, it is possible to buy exactly 13 oranges (one small pack and one large pack) or 15 oranges (five small packs) but it is not possible to buy exactly 14 oranges. What is the maximum number of oranges that cannot be bought exactly?

Problem 17. Find the positive integer k that satisfies

$$(3^3)(6^6)(15^{15})(21^{21})(k^k) = (5^5)(7^7)(10^{10})(14^{14})(27^{27}).$$

Problem 18. Determine the number of integers k with $1 \leq k \leq 2020$ for which the sum

$$1 + 2 + 3 + \cdots + k$$

is divisible by 5.

Problem 19. Given a triangle ABC. Point D is on the side BC. Point E is also on the side BC such that AE bisects $\angle CAD$. If $\angle AEB = 60°$, what is $\angle ACB + \angle ADB$ in degrees?

Problem 20. For any two positive integers a and b with the same number of digits, the operation $\langle a, b\rangle$ denotes the sum of the products of corresponding digits of a and b. For example, $\langle 1234, 5678\rangle = (1 \times 5) + (2 \times 6) + (3 \times 7) + (4 \times 8) = 70$.

Determine the value of

$$\Big\langle \underbrace{123123123\cdots}_{\text{2020 digits}}, \underbrace{123412341234\cdots}_{\text{2020 digits}} \Big\rangle.$$

6 Junior Category

Part A (1 point each)

Problem 1. A cuboid has length 3, width 3, and height 23. It has the same surface area as a cube. What is the side length of the cube?

Problem 2. If we write 2020 as a sum of 5 consecutive integers, what is the smallest integer?

Problem 3. Find the last digit of $4^4 + 5^5 + 6^6 + 7^7$.

Problem 4. The product of two positive integers M and N is 50 times their sum and 75 times their difference. Find $M + N$.

Problem 5. Determine the number of pairs of positive integers (x, y) that satisfy the inequality $5x + 3y \leq 53$.

Note: The pairs $(1, 2)$ and $(2, 1)$ are considered distinct pairs.

Part B (2 points each)

Problem 6. Given that

$$4^{4^n} = 2^{2^{3^{2^2}}}.$$

Find n.

Problem 7. Determine the greatest integer such that no digit appears more than once, and no two neighbouring digits have difference 1.

Problem 8. Given a six-digit number N. How many seven-digit numbers are there such that when we remove one of its digits, the result is N?

Problem 9. Given a quadrilateral $ABCD$ with $AB = BC = CD$, $\angle ABC = 90°$, and $\angle BCD = 60°$. Find $\angle ADC$ in degrees.

Problem 10. A math club has 7 members. The club advisor wanted to choose at least 4 members to represent the club in a math contest. How many ways are there to do so?

Part C (3 points each)

Problem 11. In a fruit shop, oranges are not sold individually; they are sold either in small packs (3 oranges) or in large packs (10 oranges). For example, it is possible to buy exactly 13 oranges (one small pack and one large pack) or 15 oranges (five small packs) but it is not possible to buy exactly 14 oranges. What is the maximum number of oranges that cannot be bought exactly?

Problem 12. Find the positive integer k that satisfies

$$(3^3)(6^6)(15^{15})(21^{21})(k^k) = (5^5)(7^7)(10^{10})(14^{14})(27^{27}).$$

Problem 13. Determine the number of integers k with $1 \leq k \leq 2020$ for which the sum

$$1 + 2 + 3 + \cdots + k$$

is divisible by 5.

Problem 14. Given a triangle ABC. Point D is on the side BC. Point E is also on the side BC such that AE bisects $\angle CAD$. If $\angle AEB = 60°$, what is $\angle ACB + \angle ADB$ in degrees?

Problem 15. For any two positive integers a and b with the same number of digits, the operation $\langle a, b \rangle$ denotes the sum of the products of corresponding digits of a and b. For example, $\langle 1234, 5678 \rangle = (1 \times 5) + (2 \times 6) + (3 \times 7) + (4 \times 8) = 70$.

Determine the value of

$$\Big\langle \underbrace{123123123\cdots}_{\text{2020 digits}}, \underbrace{123412341234\cdots}_{\text{2020 digits}} \Big\rangle.$$

Part D (4 points each)

Problem 16. Consider the following table:

$$\begin{array}{ccccccc}
1 \\
2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 & 10 \\
11 & 12 & 13 & 14 & 15 \\
\vdots & & & & & \ddots
\end{array}$$

What is the number directly below 2020?

Problem 17. The regular 20-sided polygon

$$ABCDEFGHIJKLMNPQRSTU$$

has center O. Find $\angle IMO + \angle NST$ in degrees.

Problem 18. Determine how many integers n that fulfill all the following conditions:

1. $1 \leq n \leq 1000$;
2. the digit sum of n is odd;
3. the digit sum of $(n + 1)$ is odd.

Problem 19. Which four-digit perfect square starts with two equal digits and ends with two equal digits?

Problem 20. There are 99 students (call them Student 1, Student 2, and so on) and 99 seats around a round table. How many ways are there to arrange the seating of all the students so that for any two neighboring students, their numbers differ by at most 2?

7 Senior Category

Part A (1 point each)

Problem 1. Given that

$$4^{4^n} = 2^{2^{3^{2^2}}}.$$

Find n.

Problem 2. Determine the greatest integer such that no digit appears more than once, and no two neighbouring digits have difference 1.

Problem 3. Given a six-digit number N. How many seven-digit numbers are there such that when we remove one of its digits, the result is N?

Problem 4. Given a quadrilateral $ABCD$ with $AB = BC = CD$, $\angle ABC = 90°$, and $\angle BCD = 60°$. Find $\angle ADC$ in degrees.

Problem 5. A math club has 7 members. The club advisor wanted to choose at least 4 members to represent the club in a math contest. How many ways are there to do so?

Part B (2 points each)

Problem 6. In a fruit shop, oranges are not sold individually; they are sold either in small packs (3 oranges) or in large packs (10 oranges). For example, it is possible to buy exactly 13 oranges (one small pack and one large pack) or 15 oranges (five small packs) but it is not possible to buy exactly 14 oranges. What is the maximum number of oranges that cannot be bought exactly?

Problem 7. Find the positive integer k that satisfies

$$(3^3)(6^6)(15^{15})(21^{21})(k^k) = (5^5)(7^7)(10^{10})(14^{14})(27^{27}).$$

Problem 8. Determine the number of integers k with $1 \leq k \leq 2020$ for which the sum

$$1 + 2 + 3 + \cdots + k$$

is divisible by 5.

Problem 9. Given a triangle ABC. Point D is on the side BC. Point E is also on the side BC such that AE bisects $\angle CAD$. If $\angle AEB = 60°$, what is $\angle ACB + \angle ADB$ in degrees?

Problem 10. For any two positive integers a and b with the same number of digits, the operation $\langle a, b \rangle$ denotes the sum of the products of corresponding digits of a and b. For example, $\langle 1234, 5678 \rangle = (1 \times 5) + (2 \times 6) + (3 \times 7) + (4 \times 8) = 70$.

Determine the value of

$$\Big\langle \underbrace{123123123\cdots}_{2020 \text{ digits}}, \underbrace{123412341234\cdots}_{2020 \text{ digits}} \Big\rangle.$$

Part C (3 points each)

Problem 11. Consider the following table:

$$\begin{array}{lllll} 1 & & & & \\ 2 & 3 & & & \\ 4 & 5 & 6 & & \\ 7 & 8 & 9 & 10 & \\ 11 & 12 & 13 & 14 & 15 \\ \vdots & & & & & \ddots \end{array}$$

What is the number directly below 2020?

Problem 12. The regular 20-sided polygon

$$ABCDEFGHIJKLMNPQRSTU$$

has center O. Find $\angle IMO + \angle NST$ in degrees.

Problem 13. Determine how many integers n that fulfill all the following conditions:

1. $1 \leq n \leq 1000$;
2. the digit sum of n is odd;
3. the digit sum of $(n + 1)$ is odd.

Problem 14. Which four-digit perfect square starts with two equal digits and ends with two equal digits?

Problem 15. There are 99 students (call them Student 1, Student 2, and so on) and 99 seats around a round table. How many ways are there to arrange the seating of all the students so that for any two neighboring students, their numbers differ by at most 2?

Part D (4 points each)

Problem 16. The number $\overline{2a9b} = (2 \times 1000) + (a \times 100) + (9 \times 10) + b$ is the product of two integers 2^a and 9^b. How many different values of $a + b$ are there?

Problem 17. Given a square $ABCD$. Point K divides the side AB in the ratio $AK : KB = 2 : 1$. Point L divides the diagonal AC in the ratio $AL : LC = 5 : 1$. Find $\angle KLD$ in degrees.

Problem 18. If we add 2500 to a prime p, we get a perfect square. What is p?

Problem 19. Determine the number of pairs of non-zero integers (m, n) that satisfy the following equation:

$$(m^2 + n)(m + n^2) = (m + n)^3.$$

Note: The pairs $(1, 2)$ and $(2, 1)$ are considered distinct pairs.

Problem 20. There are 31 students in a class; call them Student 1, Student 2, Student 3, and so on. A teacher writes a number on a whiteboard. Then, each of the students makes a statement about the number:

Student 1 says, "the number is divisible by 1."

Then, Student 2 says, "the number is divisible by 2."

Then, Student 3 says, "the number is divisible by 3."

⋮

Then, Student 31 says, "the number is divisible by 31."

Finally, the teacher says, "Each of you made a true statement, except for two students. Moreover, these two students made their statements consecutively."

Which student made the first false statement?

Part III

Jawapan/Answers

8 Kategori *Primary*/ Primary Category

Problem 1. 1

Problem 2. 7

Problem 3. 885

Problem 4. 11

Problem 5. 24

Problem 6. 7

Problem 7. 402

Problem 8. 0

Problem 9. 360

Problem 10. 82

Problem 11. 40

Problem 12. 9758642031

Problem 13. 63

Problem 14. 135

Problem 15. 64

Problem 16. 17

Problem 17. 18

Problem 18. 808

Problem 19. 120

Problem 20. 10098

9 Kategori *Junior* / Junior Category

Problem 1. 7

Problem 2. 402

Problem 3. 0

Problem 4. 360

Problem 5. 82

Problem 6. 40

Problem 7. 9758642031

Problem 8. 63

Problem 9. 135

Problem 10. 64

Problem 11. 17

Problem 12. 18

Problem 13. 808

Problem 14. 120

Problem 15. 10098

Problem 16. 2084

Problem 17. 189

Problem 18. 46

Problem 19. 7744

Problem 20. 198

10 Kategori *Senior* / Senior Category

Problem 1. 40

Problem 2. 9758642031

Problem 3. 63

Problem 4. 135

Problem 5. 64

Problem 6. 17

Problem 7. 18

Problem 8. 808

Problem 9. 120

Problem 10. 10098

Problem 11. 2084

Problem 12. 189

Problem 13. 46

Problem 14. 7744

Problem 15. 198

Problem 16. 1

Problem 17. 90

Problem 18. 101

Problem 19. 8

Problem 20. 16