



International Mathematical Olympiad
National Selection Test
MALAYSIA

IMONST 2 2020 JUNIOR CATEGORY

Instructions

1. The IMONST 2 paper contains six (6) essay/proof-type problems. Each problem is worth 7 points each. Integer partial scores are possible.
2. There are different papers for Primary, Junior, and Senior categories. Please make sure you have the paper for the right category.
3. The time allowed for the test is 5 hours, from 10.00 am to 3.00 pm.
4. You need:
 - (a) An internet connection to download the question paper and submit your solutions.
 - (b) A4 papers (blank, not lined) for scratch papers and final scripts.
 - (c) Pens, pencils, erasers, geometric tools (compass, ruler).
 - (d) **IMPORTANT:** A scanner to scan your scripts. If you don't have a scanner, please download a scanner app on your phone – we recommend Adobe Scan.
5. You are permitted to use a calculator, but this not essential.
6. Please use your handwriting to write the solutions. Do not type or use any word processing/typesetting software to produce your script.
7. Write your **full name, category, and school/institution** at the **top of the first page**.
8. This is an open book test. You may refer to any internet articles or printed material.
9. Work on the solutions on your own. Any form of outside help, such as discussions, communication, or assistance from another person, is not allowed.
10. Please follow the instructions on the contest page to submit your answer scans.

IMONST 2 2020 (Junior)

Problem 1. A right triangle has sides of lengths a , b , and c (c is the length of the hypotenuse). Inside the triangle, we place a circle such that it touches all three sides of the triangle.

Prove that the radius of this circle is equal to $\frac{a + b - c}{2}$.

Suatu segitiga bersudut tegak mempunyai sisi-sisi dengan panjang a , b , dan c (c ialah panjang bagi hipotenus). Dalam segitiga tersebut, kita muatkan suatu bulatan yang menyentuh ketiga-tiga sisi bagi segitiga tersebut.

Buktikan bahawa jejari bagi bulatan ini adalah bersamaan $\frac{a + b - c}{2}$.

Problem 2. Prove that for any integer $n \geq 6$, we can divide an equilateral triangle completely into n smaller equilateral triangles.

Buktikan bahawa bagi setiap integer $n \geq 6$, kita boleh membahagikan suatu segitiga sama sisi sepenuhnya kepada n segitiga sama sisi yang lebih kecil.

Problem 3. Given integers a and b such that $a^2 + b^2$ is divisible by 11. Prove that a and b are both divisible by 11.

Diberi integer a dan b dengan $a^2 + b^2$ boleh bahagi dengan 11. Buktikan bahawa kedua-dua a dan b boleh bahagi dengan 11.

Problem 4. Given a trapezium with two parallel sides of lengths m and n , where m and n are integers. Prove that it is possible to divide the trapezium into several congruent triangles.

Note: Two triangles are congruent if they have the same shape and the same size.

Diberi suatu trapezium dengan dua sisi selari yang panjangnya m dan n , dengan m dan n integer. Buktikan bahawa kita boleh membahagikan trapezium tersebut kepada beberapa segitiga yang kongruen.

Nota: Dua segitiga adalah kongruen jika kedua-duanya mempunyai bentuk dan saiz yang sama.

Problem 5. Prove that

Buktikan bahawa

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots + \frac{1}{2019} - \frac{1}{2020} = \frac{1}{1011} + \frac{1}{1012} + \frac{1}{1013} + \cdots + \frac{1}{2020}.$$

Problem 6. Find all possible integer values of n such that $12n^2 + 12n + 11$ is a 4-digit number with all 4 digits equal.

Cari semua nilai integer yang mungkin bagi n sehinggakan $12n^2 + 12n + 11$ adalah suatu nombor 4-digit dengan 4 digit yang sama.

■ END OF PAPER ■